Example:

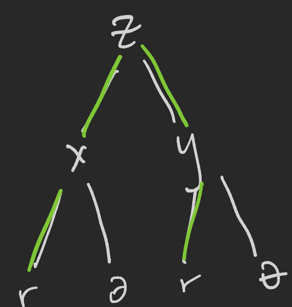
$$z = f(x,y) = \sqrt{1+xy}$$

$$x = r\cos\theta$$

$$y = r\sin\theta$$

Compute $\dfrac{\partial z}{\partial r}$ using the chain rule.

Solution:



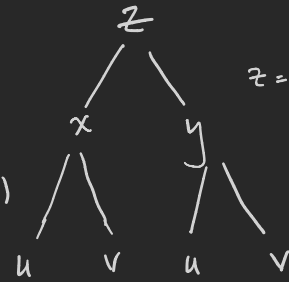
$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r}$$

$$= \frac{1}{2}(1+xy)^{-\frac{1}{2}}\, y \cos\theta$$

$$+ \frac{1}{2}(1+xy)^{-\frac{1}{2}}\, x \sin\theta$$

If wanted/needed, could express entirely in terms of r, θ.

Example:



$z = f(x,y)$

$x = g(u,v)$ $\quad$ $y = h(u,v)$

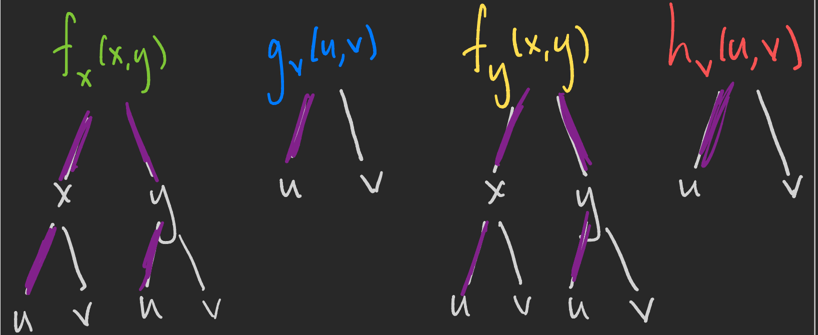
Compute $\dfrac{\partial z}{\partial u\,\partial v}$ in terms of f, g, h and their partial derivatives.

Rmk. $\dfrac{\partial z}{\partial u\,\partial v} = \dfrac{\partial}{\partial u}\left(\dfrac{\partial z}{\partial v}\right)$ (Clairaut says order doesn't really matter)

Solution:

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}$$

$$= f_x(x,y)\, g_v(u,v) + f_y(x,y)\, h_v(u,v)$$



$$\frac{\partial}{\partial u}\left(\frac{\partial z}{\partial v}\right) = \left(f_{xx}(x,y)\,g_u(u,v) + f_{xy}(x,y)\,h_u(u,v)\right) g_v(u,v) + f_x(x,y)\,g_{vu}(u,v)$$

$$+ \left(f_{yx}(x,y)\,g_u(u,v) + f_{yy}(x,y)\,h_u(u,v)\right) h_v(u,v) + f_y(x,y)\,h_{vu}(u,v)$$